A Framework for Fixed-Income Portfolio Management

The basic features of the investment management process are the same for a fixed-income portfolio as for any other type of investment. **Risk, return, and constraints are considered first.** If the client is a taxable investor, portfolio analysis must be done on an after-tax basis and considerations of the tax-efficient placement of fixed-income assets come to the fore. For any type of client, the fixed-income portfolio manager must **agree with the client on an appropriate benchmark, based on the needs of the client as expressed in the investment policy statement or the investor’s mandate to the portfolio manager.**

Broadly, there are **two types of investors based on investment objectives.**

- The first type of investor **does not have liability matching as a specific objective.** An investor not focused on liability matching will typically select a specific **bond market index as the benchmark** for the portfolio; the portfolio’s objective is to either match or exceed the rate of return on that index. This approach is sometimes referred to as **investing on a benchmark-relative basis.** However, the investor taking this approach will generally **evaluate the risk of bond holdings not only in relation to the benchmark index but also in relation to the contribution to the risk of the overall portfolio.**

- The second type of investor **has a liability (or set of liabilities) that needs to be met.** The investor with liabilities will measure success by whether the portfolio generates the funds necessary to pay out the cash outflows associated with the liabilities. In other words, **meeting the liabilities is the investment objective; as such, it also becomes the benchmark for the portfolio.**
The Fixed-Income Portfolio Management Process

1. Select the benchmark (i.e., a bond index or the client’s liability structure - see below) and specify a desired outcome relative to that benchmark.
2. Identify the risks (e.g., tracking error for an index benchmark or relevant risks for a liability structure benchmark), and
3. Specify the constraints imposed by the client, government regulators, or client's tax needs.

Manage funds against a bond market index benchmark

Select one of the following styles and strategies:

- Pure Bond Index Matching
- Enhanced Indexing/Primary Risk Factors Approach
- Enhanced Indexing/Minor Risk Factor Mismatches
- Larger Risk Factor Mismatches
- Full-Blown Active

Manage funds against a benchmark of one or more liabilities

Select one of the following strategies:

- Dedication Strategies
- Derivatives-Enabled Strategies

Passive Styles

- Immunization
- Cash Flow Matching
- Default is shown later in the reading

Active Styles

- Lower costs
- Issue selection
- Yield curve positioning
- Sector and quality positioning
- Call exposure positioning

Determine the market’s expectations (e.g., forward rates)

- Forecast the necessary inputs: e.g., changes in interest rates, changes in interest rate volatility, changes in credit spreads

Determine relative values of the securities

- Use one of the following to construct the portfolio:
  1. a cell matching technique,
  2. a multi-factor risk model

Monitor the performance of the portfolio

- Measure performance (calculate total rate of return)
- Did the active manager add value by outperforming the benchmark?
- How did the manager achieve the calculated return? (return attribution analysis)

Monitor the inputs for accuracy, given changes in the market

- Evaluate performance
- Adjust the inputs, as needed.
B- Managing Bonds against a Bond Market Index

• A **passive management strategy** assumes that the **market’s expectations are essentially correct** or, more precisely, that the manager has no reason to disagree with these expectations—perhaps because the manager has no particular expertise in forecasting. By setting the portfolio’s risk profile identical to the benchmark’s risk profile and pursuing a passive strategy, the manager is quite willing to accept an **average risk level and an average rate of return**.

• An **active management strategy** essentially relies on the manager’s forecasting ability. Active managers believe that they possess superior skills in interest rate forecasting, credit valuation, or in some other area that can be used to exploit opportunities in the market. The portfolio’s return should increase if the manager’s forecasts of the future path of the factors that influence fixed-income returns are more accurate than those reflected in the current prices of fixed-income securities. **The manager can create small mismatches (enhancement) or large mismatches (full-blown active management) relative to the benchmark to take advantage of this expertise.**

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1- **Classification of Strategies**

1) **Pure bond indexing (or full replication approach).** The goal here is to produce a portfolio that is a **perfect match to the benchmark portfolio**. The pure bond indexing approach attempts to **duplicate the index by owning all the bonds in the index in the same percentage as the index**. **Full replication is typically very difficult and expensive to implement in the case of bond indices**. Many issues in a typical bond index (particularly the non-Treasuries) are quite illiquid and very infrequently traded. For this reason, full replication of a bond index is **rarely attempted because of the difficulty, inefficiency, and high cost of implementation**.

2) **Enhanced indexing by matching primary risk factors.** This management style uses a **sampling approach in an attempt to match the primary index risk factors** and achieve a higher return than under full replication. **Primary risk factors** are typically **major influences on the pricing of bonds**, such as changes in the level of interest rates, twists in the yield curve, and changes in the spread between Treasuries and non-Treasuries.
   
   a. By investing in a **sample of bonds** rather than the whole index, the manager reduces the construction and maintenance costs of the portfolio. Although a **sampling approach will usually track the index less closely than full replication**, this disadvantage is expected to be more than offset by the lower expenses.

   b. By matching the **primary risk factors**, the portfolio is affected by broad market-moving **events** to the same degree as the benchmark index. The portfolio manager may try to enhance the portfolio’s return using **bonds that are perceived to be undervalued**, for example.

3) **Enhanced indexing by small risk factor mismatches.** While matching duration (interest rate sensitivity), this style allows the manager to tilt the portfolio in favor of any of the other risk factors. The manager may try to marginally **increase the return by pursuing relative value** in certain sectors, quality, term structure, and so on. The mismatches are small and are intended to simply
enhance the portfolio’s return enough to overcome the difference in administrative costs between the portfolio and the index.

4) **Active management by larger risk factor mismatches.** The difference between this style and enhanced indexing is one of degree. This style involves the readiness to make deliberately larger mismatches on the primary risk factors than in **Type 3.** The objective of the manager is to produce sufficient returns to overcome this style’s additional transaction costs while controlling risk.

5) **Full-blown active management.** Full-blown active management involves the possibility of aggressive mismatches on duration, sector weights, and other factors.

2- **Indexing (Pure and Enhanced)**

Several reasons exist for bond indexing.

1) **Indexed portfolios have lower fees than actively managed accounts.** Nonadvisory fees, such as custodial fees, are also much lower for indexed portfolios.

2) **Outperforming a broadly based market index on a consistent basis is a difficult task,** particularly when one has to overcome the higher fees and costs associated with active management.

3) **Broadly based bond index portfolios provide excellent diversification.** The diversification inherent in an indexed portfolio results in a lower risk for a given level of return than other less diversified portfolios.

4) **Selection of a Benchmark Bond Index: General Considerations**

The choice depends heavily on four factors:

1) **Market value risk.** The market value risk of the portfolio and benchmark index should be comparable. Given a normal upward-sloping yield curve, a bond portfolio’s yield to maturity increases as the maturity of the portfolio increases. According to the expectations theory of term structure, a rising yield curve means that investors believe interest rates will likely increase in the future. Because a long duration portfolio is more sensitive to changes in interest rates, a long portfolio will likely fall more in price than a short one. In other words, as the maturity and duration of a portfolio increase, the market risk increases. **For investors who are risk averse, the short-term or intermediate-term index may be more appropriate as a benchmark index than the long index.**

2) **Income risk:** the portfolio and benchmark should provide comparable assured income streams. If stability and dependability of income are the primary needs of the investor, then the long portfolio is the least risky and the short portfolio is the most risky.

3) **Credit risk.** The average credit risk of the benchmark index should be appropriate for the indexed portfolio’s role in the investor's overall portfolio and satisfy any constraints placed on credit quality in the investor's investment policy statement. The diversification among issuers in the benchmark index should also be satisfactory to the investor.

4) **Liability framework risk.** This risk **should be minimized.** In general, it is prudent to match the investment characteristics of assets and liabilities, if liabilities play any role.
b. Risk in Detail: Risk Profiles

The major source of risk for most bonds relates to the yield curve (the relationship between interest rates and time to maturity). Yield curve changes include:

1) A parallel shift in the yield curve (an equal shift in the interest rate at all maturities),
2) A twist of the yield curve (movement in contrary directions of interest rates at two maturities), and
3) Other curvature changes of the yield curve.

A yield curve shift typically accounts for about 90 percent of the change in value of a bond.

The risk profile of a bond index includes:

1) Interest rate risk: change in the level of interest rates measured with duration
2) Yield curve risk: changes in the shape of the yield curve captured by key rate duration and present value distribution of cash flows
3) Spread risk: changes in the spread between treasuries and non-treasuries captured with quality spread duration contributions

A completely effective indexed portfolio will have the exact same risk profile as the selected benchmark. The portfolio manager may use various techniques, perhaps in combination, to align the portfolio’s risk exposures with those of the benchmark index.

A cell-matching technique (also known as stratified sampling) divides the benchmark index into cells that represent qualities that should reflect the risk factors of the index. The manager then selects bonds from those in each cell to represent the entire cell taking account of the cell’s relative importance in the benchmark index. The total dollar amount selected from this cell may be based on that cell’s percentage of the total.

A multifactor model technique makes use of a set of factors that drive bond returns. Generally, portfolio managers will focus on the most important or primary risk factors:

1) Duration. An index’s effective duration measures the sensitivity of the index’s price to a relatively small parallel shift in interest rates (i.e., interest rate risk). (For large parallel changes in interest rates, a convexity adjustment is used to improve the accuracy of the index’s estimated price change. A convexity adjustment is an estimate of the change in price that is not explained by duration.) Because parallel shifts in the yield curve are relatively rare in isolation, duration by itself is inadequate to capture the full effect of changes in interest rates.

2) Key rate duration and present value distribution of cash flows. Nonparallel shifts in the yield curve (i.e., yield curve risk), such as an increase in slope or a twist in the curve, can be captured by two separate measures. Key rate duration is one established method for measuring the effect of shifts in key points along the yield curve. In this method, we hold the spot rates constant for all points along the yield curve but one. By changing the spot rate for that key maturity, we are able to measure a portfolio’s sensitivity to a change in that maturity. This sensitivity is called the rate duration. We repeat the process for other key points (e.g., 3 years, 7 years, 10 years, and 15 years) and measure
their sensitivities as well. Simulations of twists in the yield curve can then be conducted to see how the portfolio would react to these changes. Key rate durations are particularly useful for determining the relative attractiveness of various portfolio strategies, such as bullet strategies with maturities focused at one point on the yield curve versus barbell strategies where maturities are concentrated at two extremes. These strategies react differently to nonparallel changes in the yield curve.

Another popular indexing method is to match the portfolio’s present value distribution of cash flows to that of the benchmark. Dividing future time into a set of non-overlapping time periods, the present value distribution of cash flows is a list that associates with each time period the fraction of the portfolio’s duration that is attributable to cash flows falling in that time period. The calculation involves the following steps:

3) **Sector and quality percent.** To ensure that the bond market index’s yield is replicated by the portfolio, the manager will match the percentage weight in the various sectors and qualities of the benchmark index.

4) **Sector duration contribution.** For an indexed portfolio, the portfolio must achieve the same duration exposure to each sector as the benchmark index. The goal is to ensure that a change in sector spreads has the same impact on both the portfolio and the index.

5) **Quality spread duration contribution.** The risk that a bond’s price will change as a result of spread changes (e.g., between corporates and Treasuries) is known as spread risk. A measure that describes how a non-Treasury security’s price will change as a result of the widening or narrowing of the spread is spread duration. **Changes in the spread between qualities of bonds will also affect the rate of return. The easiest way to ensure that the indexed portfolio closely tracks the benchmark is to match the amount of the index duration that comes from the various quality categories.**

6) **Sector/coupon/maturity cell weights.** Because duration only captures the effect of small interest rate changes on an index’s value, convexity is often used to improve the accuracy of the estimated price change, particularly where the change in rates is large. However, some bonds (such as mortgage-backed securities) may exhibit negative convexity, making the index’s exposure to call risk difficult to replicate. A manager can attempt to match the convexity of the index, but such matching is rarely attempted because to stay matched can lead to excessively high transactions costs (Callable securities tend to be very illiquid and expensive to trade.). A more feasible method of matching the call exposure is to match the sector, coupon, and maturity weights of the callable sectors. As rates change, the changes in call exposure of the portfolio will be matched to the index.

7) **Issuer exposure.** Event risk is the final risk that needs to be controlled. If a manager attempts to replicate the index with too few securities, event risk takes on greater importance.

The degree of success of an indexer in mimicking the returns on a benchmark is measured by tracking risk.
Exhibit 3: Typical Fixed-Income Portfolio Risk Exposures

A Sampling of Risk Exposures in a Fixed Income Portfolio

Non MBS Securities

Interest Rate Risk
- Measures exposure to a parallel shift in the yield curve

Yield Curve Risk
- Measures exposure to a "twist" (non-parallel movement) in the yield curve

Spread Risk
- Measures exposure to changes in spreads between Treasuries and non-Treasuries

Credit Risk
- Measures exposure to downgrades and defaults

Optionality Risk
- Measures exposure to changes in cash flows due to call or put features

Mortgage Backed Securities (MBS)

Sector Risk

Prepayment Risk

Convexity Risk

Commonly used measures:
- Portfolio Duration
- Key Rate Durations
- Distribution of the PV of the cash flows
- Spread Duration
- Contribution to duration by credit risk
- Delta of the portfolio

Examples of a portfolio's "primary risk factors"
c. Tracking Risk

Tracking risk (also known as tracking error) is a measure of the variability with which a portfolio’s return tracks the return of a benchmark index. More specifically, tracking risk is defined as the standard deviation of the portfolio’s active return, where the active return for each period is defined as

\[
\text{Active return} = \text{Portfolio's return} - \text{Benchmark index's return}
\]

Therefore,

\[
\text{Tracking risk} = \text{Standard deviation of the active returns}
\]

Tracking risk arises primarily from mismatches between a portfolio’s risk profile and the benchmark’s risk profile.

d. Enhanced Indexing Strategies

Although there are expenses and transaction costs associated with constructing and rebalancing an indexed portfolio, there are no similar costs for the index itself. Therefore, it is reasonable to expect that a perfectly indexed portfolio will underperform the index by the amount of these costs. For this reason, the bond manager may choose to recover these costs by seeking to enhance the portfolio’s return. There are a number of ways (i.e., index enhancement strategies) in which this may be done:

1) Lower cost enhancements. Managers can increase the portfolio’s net return by simply maintaining tight controls on trading costs and management fees.

2) Issue selection enhancements. The manager may identify and select securities that are undervalued in the marketplace, relative to a valuation model’s theoretical value. The manager may be able to select issues that will soon be upgraded and avoid those issues that are on the verge of being downgraded.

3) Yield curve positioning. Some maturities along the yield curve tend to remain consistently overvalued or undervalued. For example, the yield curve frequently has a negative slope between 25 and 30 years, even though the remainder of the curve may have a positive slope. These long-term bonds tend to be popular investments for many institutions, resulting in an overvalued price relative to bonds of shorter maturities. By overweighting the undervalued areas of the curve and underweighting the overvalued areas, the manager may be able to enhance the portfolio’s return.

4) Sector and quality positioning. This return enhancement technique takes two forms:
   a. Maintaining a yield tilt toward short duration corporates. Experience has shown that the best yield spread per unit of duration risk is usually available in corporate securities with less than five years to maturity. A manager can increase the return on the portfolio without a commensurate increase in risk by tilting the portfolio toward these securities. The strategy is not without its risks, although these are manageable. Default risk is higher for corporate securities, but this risk can be managed through proper diversification.
   b. Periodic over- or underweighting of sectors (e.g., Treasuries vs. corporates) or qualities. Conducted on a small scale, the manager may overweight Treasuries when spreads are
expected to widen (e.g., before a recession) and underweight them when spreads are expected to narrow. Although this strategy has some similarities to active management, it is implemented on such a small scale that the objective is to earn enough extra return to offset some of the indexing expenses, not to outperform the index by a large margin as is the case in active management.

5) Call exposure positioning. A drop in interest rates will inevitably lead to some callable bonds being retired early. As rates drop, the investor must determine the probability that the bond will be called.

3- Active Strategies

An active manager is quite willing to accept a large tracking risk, with a large positive active return. By carefully applying his or her superior forecasting or analytical skills, the active manager hopes to be able to generate a portfolio return that is considerably higher than the benchmark return.

a. Extra Activities Required for the Active Manager

After selecting the type of active strategy to pursue, the active manager will:

1) Identify which index mismatches are to be exploited. If the manager’s strength is interest rate forecasting, deliberate mismatches in duration will be created between the portfolio and the benchmark. If the manager possesses superior skill in identifying undervalued securities or undervalued sectors, sector mismatches will be pursued.

2) Extrapolate the market’s expectations (or inputs) from the market data. By analyzing these prices and yields, additional data can be obtained. Forward rates can be calculated from the points along the spot rate yield curve. These forward rates can provide insight into the direction and level that investors believe rates will be headed in the future.

3) Independently forecast the necessary inputs and compare these with the market’s expectations. For example, after calculating the forward rates, the active manager may fervently believe that these rates are too high and that future interest rates will not reach these levels.

4) Estimate the relative values of securities in order to identify areas of under- or overvaluation. Some managers will make duration mismatches while others will focus on undervalued securities. In all cases, however, the managers will apply their skills to try and exploit opportunities as they arise.

b. Total Return Analysis and Scenario Analysis

In order to analyze the risk and return characteristics and the impact a trade will have on a portfolio’s return the manager can use two primary tools total return analysis and scenario analysis:

1) Total return analysis: The total return on a bond is the rate of return that equates the future value of the bond’s cash flows with the full price of the bond. As such, the total return takes into account all three sources of potential return: coupon income, reinvestment income, and change in price. Total return analysis involves assessing the expected effect of a trade on the portfolio’s total return given an interest rate forecast.
To compute total return when purchasing a bond with semiannual coupons, for example, the manager needs to specify

a) an investment horizon,
b) an expected reinvestment rate for the coupon payments, and
c) the expected price of the bond at the end of the time horizon given a forecast change in interest rates.

The manager may want to start with his prediction of the most likely change in interest rates. The semiannual total return that the manager would expect to earn on the trade is:

$$\text{Semiannual total return} = \left( \frac{\text{Total future dollars}}{\text{Full price of the bond}} \right)^{1/n} - 1$$

Where $n$ is the number of periods in the investment horizon.

A prudent manager will want to conduct a scenario analysis to evaluate the impact of the trade on expected total return under all reasonable sets of assumptions (assessing the volatility/distribution of returns under a different set of assumptions.)

2) **Scenario analysis** is useful in a variety of ways:

a) The obvious benefit is that the manager is able to assess the distribution of possible outcomes, in essence conducting a risk analysis on the portfolio’s trades. The manager may find that, even though the expected total return is quite acceptable, the distribution of outcomes (volatility) is so wide that it exceeds the risk tolerance of the client.

b) The analysis can be reversed, beginning with a range of acceptable outcomes, then calculating the range of interest rate movements (inputs) that would result in a desirable outcome. The manager can then place probabilities on interest rates falling within this acceptable range and make a more informed decision on whether to proceed with the trade.

c) The contribution of the individual components (inputs) to the total return may be evaluated. The manager’s a priori assumption may be that a twisting of the yield curve will have a small effect relative to other factors. The results of the scenario analysis may show that the effect is much larger than the manager anticipated, alerting him to potential problems if this area is not analyzed closely.

d) The process can be broadened to evaluate the relative merits of entire trading strategies.

**C- Managing Funds against Liabilities**

1- **Dedication Strategies**

Dedication strategies are specialized fixed-income strategies that are designed to accommodate specific funding needs of the investor. They generally are classified as passive in nature, although it is possible to add some active management elements to them. Immunization aims to construct a portfolio that, over a specified horizon, will earn a predetermined return regardless of interest rate changes. Another
widely used dedication strategy is **cash flow matching**, which provides the future funding of a liability stream from the coupon and matured principal payments of the portfolio.

There are four typical types (or classes) of liabilities that can be identified. **The more uncertain the liabilities, the more difficult it becomes to use a passive dedication strategy to achieve the portfolio’s goals.** So, as liabilities become more uncertain, managers often insert elements of active management. The goal of this action is to increase the upside potential of the portfolio while simultaneously ensuring a set of cash flows that are expected to be adequate for paying the anticipated liabilities.

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**a. Immunization Strategies**

Immunization is a popular strategy for “locking in” a guaranteed rate of return over a particular time horizon. As interest rates increase, the decrease in the price of a fixed-income security is usually at least partly offset by a higher amount of reinvestment income. As rates decline, a security’s price increase is usually at least partly offset by a lower amount of reinvestment income. For an arbitrary time horizon, the price and reinvestment effects generally do not exactly offset each other: The change in price may be either greater than or less than the change in reinvestment income. **The purpose of immunization is to identify the portfolio for which the change in price is exactly equal to the change in**
reinvestment income at the time horizon of interest. If the manager can construct such a portfolio, an assured rate of return over that horizon is locked in. The implementation of an immunization strategy depends on the type of liabilities that the manager is trying to meet: a single liability (e.g., a guaranteed investment contract), multiple liabilities (a defined-benefit plan’s promised payouts), or general cash flows (where the cash flows are more arbitrary in their timing).

**i. Classical Single-Period Immunization**

Classical immunization can be defined as the creation of a fixed-income portfolio that produces an assured return for a specific time horizon, irrespective of any parallel shifts in the yield curve. In its most basic form, the important characteristics of immunization are:

i. Specified time horizon.

ii. Assured rate of return during the holding period to a fixed horizon date.

iii. Insulation from the effects of interest rate changes on the portfolio value at the horizon date.

The fundamental mechanism supporting immunization is a portfolio structure that balances the change in the value of the portfolio at the end of the investment horizon with the return from the reinvestment of portfolio cash flows (coupon payments and maturing securities). That is, immunization requires offsetting price risk and reinvestment risk. To accomplish this balancing requires the management of duration. Setting the duration of the portfolio equal to the specified portfolio time horizon assures the offsetting of positive and negative incremental return sources under certain assumptions, including the assumption that the immunizing portfolio has the same present value as the liability being immunized. Duration-matching is a minimum condition for immunization.

Keep in mind that to immunize a portfolio’s target value or target yield against a change in the market yield, a manager must invest in a bond or a bond portfolio whose 1) duration is equal to the investment horizon and 2) initial present value of all cash flows equals the present value of the future liability.

**ii. Rebalancing an Immunized Portfolio**

The market yield will fluctuate over the investment horizon, as a result, the duration of the portfolio will change as the market yield changes. The duration will also change simply because of the passage of time. In any interest rate environment that is different from a flat term structure, the duration of a portfolio will change at a different rate from time. How often should a portfolio be rebalanced to adjust its duration? The answer involves balancing the costs and benefits of rebalancing.

**iii. Determining the Target Return**

Given the term structure of interest rates or the yield curve prevailing at the beginning of the horizon period, the assured rate of return of immunization can be determined. Theoretically, this immunization target rate of return is defined as the total return of the portfolio, assuming no change in the term structure. This target rate of return will always differ from the portfolio’s present yield to maturity unless the term structure is flat (not increasing or decreasing), because by virtue of the passage of time, there is a return effect as the portfolio moves along the yield curve (matures). That is, for an upward-sloping yield curve, the yield to maturity of a portfolio can be quite different from its immunization target rate of return while, for a flat yield curve, the yield to maturity would roughly
approximate the assured target return. In general, for an upward-sloping yield curve, the immunization target rate of return will be less than the yield to maturity because of the lower reinvestment return. Conversely, a negative or downward-sloping yield curve will result in an immunization target rate of return greater than the yield to maturity because of the higher reinvestment return.

- Alternative measures of the immunization target rate of return include the yield implied by a zero coupon bond of quality and duration comparable with that of the bond portfolio and an estimate based on results of a simulation that rebalances the initial portfolio, given scenarios of interest rate change.
- The most conservative method for discounting liabilities—the method resulting in the largest present value of the liabilities—involves the use of the Treasury spot curve (the term structure of Treasury zero coupon bonds).
- A more realistic approach utilizes the yield curve (converted to spot rates) implied by the securities held in the portfolio. This yield curve can be obtained using a curve-fitting methodology. Because spreads may change as well as the term structure itself, the value of the liabilities will vary over time.

iv. Time Horizon
The immunized time horizon is equal to the portfolio duration. Portfolio duration is equal to a weighted average of the individual security durations where the weights are the relative amounts or percentages invested in each.

v. Dollar Duration and Controlling Positions

Dollar duration is a measure of the change in portfolio value for a 100 bps change in market yields. It is defined as:

\[
\text{Dollar duration} = \text{Duration} \times \text{portfolio value} \times 0.01
\]

A portfolio’s dollar duration is equal to the sum of the dollar durations of the component securities.

In a number of ALM applications, the investor’s goal is to reestablish the dollar duration of a portfolio to a desired level. This rebalancing involves the following steps:

i. Move forward in time and include a shift in the yield curve. Using the new market values and durations, calculate the dollar duration of the portfolio at this point in time.

ii. Calculate the rebalancing ratio by dividing the original dollar duration by the new dollar duration. If we subtract one from this ratio and convert the result to a percent, it tells us the percentage amount that each position needs to be changed in order to rebalance the portfolio.

iii. Multiply the new market value of the portfolio by the desired percentage change in Step 2. This number is the amount of cash needed for rebalancing.

vi. Spread Duration
Spread duration is a measure of how the market value of a risky bond (portfolio) will change with respect to a parallel 100 bps change in its spread above the comparable benchmark security.
Spread duration is an important measurement tool for the management of spread risk. Spreads do change and the portfolio manager needs to know the risks associated with such changes.

A characteristic of bonds with credit risk (risk of loss because of credit events such as default or downgrades in credit ratings)—sometimes called “spread product”—is that their yield will be higher than a comparable risk-free security. The large spectrum of bond products available in the marketplace leads to differing types of spread duration. The three major types are:

i. **Nominal spread**, the spread of a bond or portfolio above the yield of a certain maturity Treasury.

ii. **Static spread or zero-volatility spread**, defined as the constant spread above the Treasury spot curve that equates the calculated price of the security to the market price.

iii. **Option-adjusted spread (OAS)**, the current spread over the benchmark yield minus that component of the spread that is attributable to any embedded optionality in the instrument.

The spread duration of a portfolio is calculated as a market weighted average of the spread durations of the component securities. For a portfolio of non-Treasury securities, spread duration equals portfolio duration. However, because the spread duration of Treasury securities is zero, a portfolio that includes both Treasury and non-Treasury securities will have a spread duration that is different from the portfolio duration.

**Note: Portfolio Immunization**

1) A great thing about immunization is that it is a set-and-forget strategy. That is, once you have immunized your portfolio, there is no subsequent work to be done. This statement is incorrect. One needs to rebalance the portfolio duration whenever interest rates change and as time elapses since the previous rebalancing.

2) The immunization target rate of return is less than yield to maturity. This statement is only true if the yield curve is upward sloping. If the yield curve is downward-sloping, then this statement is not true as the immunization target rate of return would exceed the yield to maturity because of the higher reinvestment return.

3) If a portfolio is immunized against a change in the market yield at a given horizon by matching portfolio duration to horizon, the portfolio faces no risk except for default risk. The statement is incorrect. The portfolio described would be exposed to the risk of a change in interest rates that results in a change in the shape of the yield curve.

4) The liquidity of securities used to construct an immunized portfolio is irrelevant. The statement is incorrect because immunized portfolios need to be rebalanced; the liquidity of securities used to construct an immunized portfolio is a relevant consideration. Illiquid securities involve high transaction costs and make portfolio rebalancing costly.
5) In general, the entire portfolio does not have to be turned over to rebalance an immunized portfolio. Furthermore, rebalancing need not be done on a daily basis. The statement is correct. The entire portfolio does not have to be turned over to rebalance it because shifting a small set of securities from one maturity range to another is generally enough. Also, to avoid excessive transactions costs, rebalancing is usually not done on a daily basis, which could involve excessive transaction costs.

b. Extensions of Classical Immunization Theories

Classical immunization theory is based on several assumptions:

1) Any changes in the yield curve are parallel changes, that is, interest rates move either up or down by the same amount for all maturities.
2) The portfolio is valued at a fixed horizon date, and there are no interim cash inflows or outflows before the horizon date.
3) The target value of the investment is defined as the portfolio value at the horizon date if the interest rate structure does not change (i.e., there is no change in forward rates).

Perhaps the most critical assumption of classical immunization techniques is the first one concerning the type of interest rate change anticipated. A property of a classically immunized portfolio is that the target value of the investment is the lower limit of the value of the portfolio at the horizon date if there are parallel interest rate changes. According to the theory, if there is a change in interest rates that does not correspond to this shape-preserving shift, matching the duration to the investment horizon no longer assures immunization. Non-shape-preserving shifts are the commonly observed case.
A natural extension of classical immunization theory is to extend the theory to the case of nonparallel shifts in interest rates. Two approaches have been taken.

i. The first approach has been to modify the definition of duration so as to allow for nonparallel yield curve shifts, such as multifunctional duration (also known as functional duration or key rate duration).

ii. The second approach is a strategy that can handle any arbitrary interest rate change so that it is not necessary to specify an alternative duration measure. This approach establishes a measure of immunization risk against any arbitrary interest rate change. The immunization risk measure can then be minimized subject to the constraint that the duration of the portfolio equals the investment horizon, resulting in a portfolio with minimum exposure to any interest rate movements.

A second extension of classical immunization theory applies to overcoming the limitations of a fixed horizon. Under the assumption of parallel interest rate changes, a lower bound exists on the value of an investment portfolio at any particular time, although this lower bound may be below the value realized if interest rates do not change. Multiple liability immunization involves an investment strategy that guarantees meeting a specified schedule of future liabilities, regardless of the type of shift in interest rate changes.

The third extension of classical immunization theory is to analyze the risk and return trade-off for immunized portfolios with an approach called “return maximization.”

The fourth extension of classical immunization theory is to integrate immunization strategies with elements of active bond portfolio management strategies. The traditional objective of immunization has been risk protection, with little consideration of possible returns. Contingent immunization, which provides a degree of flexibility in pursuing active strategies while ensuring a certain minimum return in the case of a parallel rate shift. In contingent
immunization, immunization serves as a fall-back strategy if the actively managed portfolio does not grow at a certain rate. Contingent immunization is possible when the prevailing available immunized rate of return is greater than the required rate of return. The difference between the minimum acceptable return and the higher possible immunized rate is called the cushion spread.

i. Duration and Convexity of Assets and Liabilities

In order for a manager to have a clear picture of the economic surplus of the portfolio—defined as the market value of assets minus the present value of liabilities—the duration and convexity of both the assets and liabilities must be understood. Focusing only on the duration of a company’s assets will not give a true indication of the total interest rate risk for a company.

Convexity also plays a part in changes in economic surplus. If liabilities and assets are duration matched but not convexity matched, economic surplus will be exposed to variation in value from interest rate changes reflecting the convexity mismatch.

ii. Types of Risk

As the market environment changes, the portfolio manager faces the risk of not being able to pay liabilities when they come due. Three sources of this risk are interest rate risk, contingent claim risk, and cap risk.

1) Interest rate risk. Because the prices of most fixed-income securities move opposite to interest rates, a rising interest rate environment will adversely affect the value of a portfolio. If assets need to be sold to service liabilities, the manager may find a shortfall. Interest rate risk is the largest risk that a portfolio manager will face.

2) Contingent claims risk. In a falling rate environment, the manager may have lucrative coupon payments halted and receive principal (as is the case with mortgage-backed securities when the underlying mortgages prepay principal). The loss of the coupons is bad enough but now the principal must be reinvested at a lower rate. In addition, the market value of a callable security will level out at the call price, rather than continuing upwards as a noncallable security would.

3) Cap risk. An asset that makes floating rate payments will typically have caps associated with the floating rate. The manager is at risk of the level of market rates rising while the asset returns are capped. This event may severely affect the value of the assets.

iii. Risk Minimization for Immunized Portfolios

An extension of classical immunization theory produced an immunized portfolio with a minimum exposure to any arbitrary interest rate change. It should be clear that reinvestment risk determines immunization risk. The portfolio that has the least reinvestment risk will have the least immunization risk. When there is a high dispersion of cash flows around the horizon date, as in the barbell portfolio, the portfolio is exposed to high reinvestment risk. When the cash flows are concentrated around the horizon date, as in the bullet portfolio, the portfolio is subject to minimal reinvestment risk. In the case of a pure discount instrument maturing at the investment horizon, immunization risk is zero because, with no interim cash flows, reinvestment risk is absent. Moving from pure discount instruments to coupon payment instruments, the portfolio manager is confronted with the task of selecting coupon-
paying securities that provide the lowest immunization risk—if the manager can construct a portfolio that replicates a pure discount instrument that matures at the investment horizon, immunization risk will be zero.

Recall that the target value of an immunized portfolio is a lower bound on the terminal value of the portfolio at the investment horizon if yields on all maturities change by the same amount. If yields of different maturities change by different amounts, the target value is not necessarily the lower bound on the investment value.

If forward rates change by any arbitrary function, the relative change in the portfolio value depends on the product of two terms. The first term, denoted $M^2$ depends solely on the structure of the investment portfolio, while the second term is a function of interest rate movement only. The second term characterizes the nature of the interest rate shock. It is an uncertain quantity and, therefore, outside the control of the manager. The first term, however, is under the control of the manager, as it depends solely on the composition of the portfolio. The first term can be used as a measure of immunization risk because when it is small, the exposure of the portfolio to any interest rate change is small. The immunization risk measure $M^2$ is the variance of time to payment around the horizon date, where the weight for a particular time in the variance calculation is the proportion of the instrument's total present value that the payment received at that time represents. The immunization risk measure may be called the maturity variance; in effect, it measures how much a given immunized portfolio differs from the ideal immunized portfolio consisting of a single pure discount instrument with maturity equal to the time horizon.

Given the measure of immunization risk that is to be minimized and the constraint that the duration of the portfolio equals the investment horizon, the optimal immunized portfolio can be found using linear programming (optimization in which the objective function and constraints are linear). Linear programming is appropriate because the risk measure is linear in the portfolio payments.

The immunization risk measure can be used to construct approximate confidence intervals for the target return over the horizon period and the target end-of-period portfolio value. A confidence interval represents an uncertainty band around the target return within which the realized return can be expected with a given probability. The expression for the confidence interval is:

\[
\text{Confidence interval} = \text{Target return} \pm (k) \times (\text{Standard deviation of target return})
\]

where $k$ is the number of standard deviations around the expected target return. The desired confidence level determines $k$. The higher the desired confidence level, the larger $k$, and the wider the band around the expected target return.

The standard deviation of the expected target return can be approximated by the product of three terms:

1) the immunization risk measure,
2) the standard deviation of the variance of the one-period change in the slope of the yield curve, and
3) an expression that is a function of the horizon length only.
c. Multiple Liability Immunization

More often, there are a number of liabilities to be paid from the investment funds and no single horizon that corresponds to the schedule of liabilities. A portfolio is said to be immunized with respect to a given liability stream if there are enough funds to pay all the liabilities when due, even if interest rates change by a parallel shift.

The conditions that must be satisfied to assure multiple liability immunization in the case of parallel rate shifts. The necessary and sufficient conditions are:

1. The present value of the assets equals the present value of the liabilities.
2. The (composite) duration of the portfolio must equal the (composite) duration of the liabilities.
3. The distribution of durations of individual portfolio assets must have a wider range than the distribution of the liabilities.

Just as in the single investment horizon case, the relative change in the portfolio value if forward rates change by any arbitrary function depends on the product of two terms: a term solely dependent on the structure of the portfolio and a term solely dependent on the interest rate movement. An optimal immunization strategy is to minimize the immunization risk measure subject to the constraints imposed by these two conditions (and any other applicable portfolio constraints). Constructing minimum-risk immunized portfolios can then be accomplished by the use of linear programming.

Approximate confidence intervals can also be constructed in the multiple liability case. The standard deviation of the expected target return is the product of the three terms indicated in the section on risk minimization.

d. Immunization for General Cash flows

What if, a given schedule of liabilities to be covered by an immunized investment must be met by investment funds that are not available at the time the portfolio is constructed?

Under certain conditions, such a strategy is indeed possible. The expected cash contributions can be considered the payments on hypothetical securities that are part of the initial holdings. The actual initial investment can then be invested in such a way that the real and hypothetical holdings taken together represent an immunized portfolio.

For example: at the end of the first year, any decline in the interest rates at which the cash contribution is invested will be offset by a corresponding increase in the value of the initial holdings. The portfolio is at that time rebalanced by selling the actual holdings and investing the proceeds together with the new cash in a portfolio with a duration of 1 to match the horizon date. Note that the rate of return guaranteed on the future contributions is not the current spot rate but rather the forward rate for the date of contribution. This strategy can be extended to apply to multiple contributions and liabilities, which produces a general immunization technique that is applicable to the case of arbitrary cash flows over a period. The construction of an optimal immunized portfolio involves quantifying and then
minimizing the immunization risk measure. Linear programming methods can then be used to obtain the optimal portfolio.

e. Return Maximization for Immunized Portfolios

The objective of risk minimization for an immunized portfolio may be too restrictive in certain situations. If a substantial increase in the expected return can be accomplished with little effect on immunization risk, the higher-yielding portfolio may be preferred in spite of its higher risk.

The **required terminal value, plus a safety margin in money terms, will determine the minimum acceptable return over the horizon period.** The difference between the minimum acceptable return and the higher possible immunized rate is known as the cushion spread. This spread offers the manager latitude in pursuing an active strategy. The greater the cushion spread, the more scope the manager has for an active management policy.

The **risk/return trade-off approach for immunized portfolios maintains the duration of the portfolio at all times equal to the horizon length.** Thus, the portfolio stays fully immunized in the classical sense. Instead of minimizing the immunization risk against nonparallel rate changes, however, a trade-off between risk and return is considered. The immunization risk measure can be relaxed if the compensation in terms of expected return warrants it. Specifically, the strategy maximizes a lower bound on the portfolio return. The lower bound is defined as the lower confidence interval limit on the realized return at a given confidence level.

**Linear programming can be used to solve for the optimal portfolio when return maximization is the objective.** In fact, parametric linear programming can be employed to determine an efficient frontier for immunized portfolios analogous to those in the mean–variance framework.

### 2- Cash Flow Matching Strategies

Cash flow matching is an alternative to multiple liability immunization in asset/liability management. **Cash flow matching is an appealing strategy because the portfolio manager need only select securities to match the timing and amount of liabilities.** Conceptually, a bond is selected with a maturity that matches the last liability, and an amount of principal equal to the amount of the last liability minus the final coupon payment is invested in this bond. The remaining elements of the liability stream are then reduced by the coupon payments on this bond, and another bond is chosen for the next-to-last liability, adjusted for any coupon payments received on the first bond selected. Going back in time, this sequence is continued until all liabilities have been matched by payments on the securities selected for the portfolio. Linear programming techniques can be employed to construct a least-cost cash flow matching portfolio from an acceptable universe of bonds.

**a. Cash Flow Matching versus Multiple Liability Immunization**

If all the liability flows were perfectly matched by the asset flows of the portfolio, the resulting portfolio would have no reinvestment risk and, therefore, no immunization or cash flow match risk. Given typical liability schedules and bonds available for cash flow matching, however, perfect matching is unlikely.
Under such conditions, a minimum immunization risk approach should be as good as cash flow matching and likely will be better, because an immunization strategy would require less money to fund liabilities. Two factors contribute to this superiority.

1) First, cash flow matching requires a relatively conservative rate of return assumption for short-term cash and cash balances may be occasionally substantial. By contrast, an immunized portfolio is essentially fully invested at the remaining horizon duration.

2) Second, funds from a cash flow–matched portfolio must be available when (and usually before) each liability is due, because of the difficulty in perfect matching. Because the reinvestment assumption for excess cash for cash flow matching extends many years into the future, a conservative interest rate assumption is appropriate. An immunized portfolio needs to meet the target value only on the date of each liability, because funding is achieved by a rebalancing of the portfolio.

Thus, even with the sophisticated linear programming techniques used, in most cases cash flow matching will be technically inferior to immunization. Cash flow matching is easier to understand than multiple liability immunization, however; this ease of use occasionally supports its selection in dedication portfolio strategies.

b. Extensions of Basic Cash Flow Matching

In basic cash flow matching, only asset cash flows occurring prior to a liability date can be used to satisfy the liability. The basic technique can be extended to allow cash flows occurring both before and after the liability date to be used to meet a liability. This technique, called symmetric cash flow matching, allows for the short-term borrowing of funds to satisfy a liability prior to the liability due date. The opportunity to borrow short-term so that symmetric cash matching can be employed results in a reduction in the cost of funding a liability.

A popular variation of multiple liability immunization and cash flow matching to fund liabilities is one that combines the two strategies. This strategy, referred to as combination matching or horizon matching, creates a portfolio that is duration-matched with the added constraint that it be cash flow–matched in the first few years, usually the first five years. The advantage of combination matching over multiple liability immunization is that liquidity needs are provided for in the initial cash flow-matched period. Also, most of the curvature of yield curves is often at the short end (the first few years). Cash flow matching the initial portion of the liability stream reduces the risk associated with nonparallel shifts of the yield curve. The disadvantage of combination matching over multiple liability immunization is that the cost to fund liabilities is greater.

c. Application Considerations

i. Universe Considerations

Selection of the universe for construction of a single period immunized portfolio or a dedicated portfolio is extremely important.
• The lower the **quality of the securities considered**, the higher the potential risk and return.
  Dedication assumes that there will be no defaults, and immunization theory further assumes that securities are responsive only to overall changes in interest rates. The lower the quality of securities, the greater the probability that these assumptions will not be met.
• Further, **securities with embedded options such as call options or prepayments options (e.g., mortgage-backed securities)** complicate and may even prevent the accurate measurement of cash flow, and hence duration, frustrating the basic requirements of immunization and cash flow matching.
• Finally, **liquidity** is a consideration for immunized portfolios, because they must be rebalanced periodically.

**ii. Optimization**

For an **immunized portfolio**, optimization typically takes the form of minimizing maturity variance subject to the constraints of matching weighted average duration and having the necessary duration dispersion (in multiple-liability immunization). **For cash flow matching, optimization takes the form of minimizing the initial portfolio cost subject to the constraint of having sufficient cash at the time a liability arises.** Further considerations such as average quality, minimum and maximum concentration constraints, and, perhaps, issuer constraints may be included. **Accurate pricing** is important because optimization is very sensitive to the prices of the securities under consideration. Because there are many inputs and variations available, the optimization process should be approached iteratively, with a final solution that is the result of a number of trials.

**iii. Monitoring**

Monitoring an immunized or cash flow–matched portfolio **requires periodic performance measurement.** For a bullet portfolio, performance monitoring may take the form of **regular observations of the return to date linked with the current target return and annualized.** This return should fluctuate only slightly about the original target return.

The performance of a multiple liability immunized plan can be monitored most easily by comparing the current market value of the assets with the present value of the remaining liabilities. **The current internal rate of return on the immunized portfolio should be used to discount the remaining liabilities.** (This rate is used because it is the expected rate of return that is necessary to provide sufficient cash flow to fund the liabilities.) These two quantities should track one another closely. It may also be useful to monitor the estimated standard deviation of the terminal value of the fund to make sure that it falls more or less uniformly to zero as the horizon date approaches.

**iv. Transaction Costs**

Transactions costs are important in meeting the target rate for an immunized portfolio. They must be considered not only in the initial immunization (when the immunized portfolio is first created) but also in the periodic rebalancing necessary to avoid duration mismatch.